

## DERIVATIVE :

### GROUP-A

(Each question carry 2 mark)

1. What is the derivative of  $\sin^{-1}(3x-4x^3)$  w.r.t.  $\sin^{-1}x$  ?
2. If the derivative of a function  $f$  is twice the function, then find  $f(x)$  ?
3. What is the derivative of  $\tan^{-1} x^2$  w.r.t.  $x$  ?
4. What is the derivative of  $\log x^2$  w.r.t.  $x$  ?
5. Write down the value of  $\frac{d}{dx} |\sin x|_{x \in (0, \pi/2)}$  ?
6. Write down the value of  $\frac{d}{dx} ([x] + |x|)$ ,  $-1 < x < 0$  ?
7. Give an example of a function which is continuous but not differentiable at two points?
8. Give example of two functions which are not differentiable but their difference is differentiable?
9. Give examples of a function which is even but its derivative is odd?
10. What is the derivative of  $\tan^{-1} x$  w.r.t.  $\sin^{-1} x$  ?
11. Give example of function which is continuous but not differentiable at  $x = -1$  ?
12. What is the derivative of  $|3x-1|$  w.r.t.  $x$  for  $x < 1/3$  ?
13. What is the derivative of  $\sin^{-1} \frac{2x}{1+x^2}$  w.r.t.  $\cos^{-1} \frac{1-x^2}{1+x^2}$  ?
14. What is the derivative of  $\sec^{-1} x$  w.r.t.  $x$  if  $x < -1$  ?
15. Differentiate  $\cos^{-1}(\sin x)$  w.r.t.  $x$  ?
16. Differentiate  $\sin^{-1}(\cos x)$  w.r.t.  $x$  ?
17. Find  $\frac{d}{dx} \left( \frac{y^2}{z} \right)$ ,  $y = 10^x$ ,  $z = 100^{x/2}$  ?
18. Find  $dg/df$  where  $g(x) = x^5$ ,  $f(x) = x^2$  ?
19. Find minimum value of  $n$  such that  $\frac{d^n}{dx^n} x^{100} = 0$  ? Ans: 101
20. Find maximum value of positive integer  $n$  such that  $\frac{d^{100}}{dx^{100}} x^n = 0$  ? Ans: 101
21. Find  $\frac{d}{dx} e^{[x]\ln(x+1)}$ ;  $4 \leq x < 5$  ?
22. Find minimum value of  $n$  such that  $\sum_{j=1}^n \frac{d^j}{dx^j} (\sin x) = 0$  for all  $x \in \mathbb{R}$  ?
23. If  $f$  is twice differentiable  $f''(x) = -f(x)$ ,  $f'(x) = g(x)$  and  $h(x) = f^2(x) + \{f'(x)\}^2$ ,  $h(5)=11$ , find  $h(10)$  ?

### Group-B

(Each question carry 5 mark)

1. Find  $\frac{dy}{dx}$  if  $y = 10^{\ln \sin x}$  ?

2. Prove that  $y = \tan(x+y) \Rightarrow \frac{dy}{dx} = \frac{1+y^2}{-y^2}$  ?
3. Differentiate w.r.t.  $x$  :  $\sqrt{\frac{1+\cos x}{1-\cos x}}$  ?
4. Find the value of  $\frac{d}{dx}((\tan^{-1} x)^{\sin x})$  ?
5. Prove that every differentiable function is continuous?
6. Is the following function differentiable? If no justify if  $f(x)=|3x-7|$  ?
7. If  $y=x^y$ , find  $dy/dx$ ?
8. Write why the function  $\sin^{-1} \frac{1}{\sqrt{1-x^2}}$  can not be differentiated any where?
9. Find  $\frac{dy}{dx}$  if  $\log_{(e^2)} 3$  ?

### Group-C

(Each question carry 6 mark)

9. Find  $dy/dx$  : (a)  $y = \cos^{-1} x / [1 - \sin^{-1} x]$  (b)  $y^2 = x^{\sin y}$  (c)  $y = e^{\frac{1-x}{1+x}}$
10. Find  $dy/dx$  : (a)  $e^x \cdot \tan^{-1} y + e^y \cdot \tan^{-1} x = 2$  (b)  $y = \log \tan^{-1}(e^x)$  (c)  $x^m y^n = (x+y)^{m+n}$
11. Find  $dy/dx$  : (a)  $e^{\sin(x+y)} = \cos(x+y)$  (b)  $x = e^{(1-\cos t)}$ ,  $y = e^{(1+\sin t)}$
12. Find  $dy/dx$  : (a)  $x^3 + y^3 + xy^2 + y - 1 = 0$  (b)  $y = x^{\sin x} + (\tan x)^x$  (c)  $y = \sqrt{\sin \sqrt{x}}$
13. Find  $dy/dx$  if  $x^y = y^x + \tan^{-1} \frac{\cos x}{1 + \sin x}$

14. Differentiate the following function w.r.t.  $x$  :  $\frac{x^{1/2}(1-2x)^{2/3}}{(x-1)^2(x-4)^3} + \tan^{-1} \left( \frac{\cos x - \sin x}{\cos x + \sin x} \right)$

15. (a)  $y = \tan^{-1} \sqrt{\frac{1+\sin x}{1-\sin x}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}$  (b) Find  $\frac{dy}{dx}$  if  $y = \cot^{-1}(\ln \cos e^{-x})$

16. Prove  $x^p y^q = (x+y)^{p+q} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$

### SUCCESSIVE DERIVATIVE :

#### Group-A

(Each question carry 2 mark)

1. Find the value of  $\frac{d^5}{dx^5}(\sin 2x)$ ?
2. Find the value of  $\frac{\frac{d^m}{dx^m}(x^m)}{\frac{d^{m-1}}{dx^{m-1}}(x^{m-1})}$ ?
3. What is the 21st derivative of  $(c-3x)^m$ ,  $m > 21$ ?
4. Find the 5th derivative of  $\sin x$  ?
5. What is the least value of  $n$  such that



$\frac{d^n}{dx^n} (1+x+x^2)^m = 0$  where m is a non negative integer?

Ans :  $2m + 1$

**Group-B** (Each question carry 5 mark)

6. Find  $y_n$  when  $y = \cos^3 x$ ?
7. Find  $y_4$  when  $y = 1/x (x+1)$  ?
8. Determine  $y_n(0)$ , if  $y = e^{3x} \sin(4x+2)$ ?

**Group-C** (Each question carry 10 mark)

9. Find the nth derivative of  $x^2 \sin(ax-bx)$  w.r.t. x.
10. If  $y = \sin t$ ,  $x = \sin 2t$ , prove that  $(1-y^2) \frac{d^2 x}{dy^2} - y \frac{dx}{dy} + 4x = 0$ ?
11. If  $y = e^m \cos^{-1} x$ , prove that  $(1-x^2)y_2 - xy_1 - m^2 y = 0$
12. If  $y = e^a \sin^{-1} x$ , then show that  $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$   
Find  $y_6(0)$ , if  $a = 3$  &  $y_2(0) = 9$ .
13. If  $y = \ln(x + \sqrt{1+x^2})$ , Show that  $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2 y_n = 0$ ?
14. Determine  $y_n$  when  $y = e^{ax} \sin bx$ . Deduce the value of  $y_5(0)$  taking  $y = e^{\sqrt{3}x} \sin 3x$  ?
15. (i) If  $y = e^m \tan^{-1} x$  then prove that  $(1+x^2)y_{n+1} + (2nx-m)y_n + n(n-1)y_{n-1} = 0$  ?  
(ii) If  $2z = x \left( 2 + \frac{dz}{dx} \right)$ , prove that  $\frac{d^2 z}{dx^2}$  is a constant?
16. (i) Find the nth derivative of  $x^2 \cos(b-ax)$  w.r.t. x?  
(ii) If  $x = \sin \theta$ ,  $y = \sin 5\theta$ , prove that  $(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + 25y = 0$ ?
17. If  $y = e^m \cos^{-1} x$ , then show that  $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$ .
18. If  $y = e^m \sin^{-1} x$ , then show that  $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (n^2+m^2)y_n = 0$ .
19. If  $g(x) = x^3 f(x)$ ,  $g'''(1) = 20$ ,  $f(1) = 2$ ,  $f'(1) = 1$  &  $f''(1) = 0$ , then find the value of  $f'''(1)$ .
20. If  $y = \sin(m \sin^{-1} x)$  then prove that  $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$ ?
21. If  $y = \cos(m \sin^{-1} x)$  then prove that  $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$ ?

**PARTIAL DERIVATIVE & L'HOSPITAL RULE :**

**Group-A** (Each question carry 2 mark)

- Let  $z = f(x, y)$ . Then define the partial derivative of z w.r.t. y at the point  $(a, b) \in \text{dom} f$ ?
- If  $f(x, y) = y^x$ , then find the value of  $f_x$ ?
- State with reason whether the following is true or false.

If  $f(x, y) = 2x^3 + 2y^2 + 1$ , then  $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$  at  $(-1, -1)$ ?

What is the value of  $\frac{\partial^2 u}{\partial x^2}$  if  $u = 10xy^3 e^x$ ?

If  $u = e^{xy}$ , what is the value of  $y \frac{\partial u}{\partial x}$ ?

6. If  $f(x,y) = 1/(x+y)$ , then write the degree of homogeneous function  $\frac{\partial f}{\partial x}$ ?
7. If  $f(x,y) = (x-y)/(x+y)$ , then write the degree of homogeneous function  $\frac{\partial f}{\partial x}$ ?
8. Evaluate  $\lim_{x \rightarrow 0} \frac{\sin 4x}{\tan 7x}$ ?
9. Rectify the mistakes if any:  $\lim_{x \rightarrow 1} \frac{x^3 + 3x - 4}{2x^2 + x - 3} = \lim_{x \rightarrow 1} \frac{3x^2 + 3}{4x + 1} = \lim_{x \rightarrow 1} \frac{6x}{4} = \frac{3}{2}$ ?

### Group-B

(Each question carry 5 mark)

1. If  $u = \frac{y}{x} \cos^{-1}\left(\frac{x}{y}\right) + \frac{x}{y} \cot^{-1}\left(\frac{y}{x}\right)$ , find  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ ?
2. Verify euler's theorem for  $f(x,y) = \frac{x^4}{y} + \frac{y^4}{x} + y^3$ ?
3. With  $v = f(y/x)$ , prove that  $x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} = 0$ ?
4. If  $u$  is a homogeneous function of  $x$  &  $y$  of degree 3 &  $u(1,1)=2$ , then find the value of  $u_{xx}(1,1) + 2u_{xy}(1,1) + u_{yy}(1,1)$
5. If  $u = \log(x^3 + y^3 + z^3 - 3xyz)$ , then find  $(x + y + z) \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} \right)$ ?
6. If  $u = x/y + y/z + z/x$ , show that  $\left( x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} \right) = 0$ ?
7. If  $f(x,y) = \ln(x^2 + y^2)$ . Prove that  $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$ ?
8. If  $z = y + f(v)$  where  $v = x/y$ , then evaluate  $v \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}$ ?
9. If  $u(x,y) = \frac{f(x,y)}{g(x,y)}$  and  $f(x,y)$  and  $g(x,y)$  are homogeneous function of degree 3 and 4 respectively and  $u(2,3) = -2$ , then find the value of  $u_x(2,3) + (3/2) u_y(2,3)$
10. If  $z = y + f(v)$  where  $v = x/y$ , then evaluate  $v \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}$ ?
11. Evaluate  $\lim_{x \rightarrow \frac{\pi}{2}} (\sec x - \tan x)$ ?

### Group-C

(Each question carry 10 mark)

1. Verify euler's theorem for  $z = \sqrt{y^2 - x^2} \sin^{-1}\left(\frac{y}{x}\right)$ ?
2. Verify euler's theorem if  $z = \frac{x^{1/4} - y^{1/4}}{2x^3 + 3y^3}$ ?



3. If  $z = x \cdot f(y/x)$ , show that  $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z$ ?
4. (i)  $\lim_{x \rightarrow 0^+} (\tan x)^{1/\ln x}$  (ii)  $\lim_{x \rightarrow 1} \left( \frac{1}{x-1} - \frac{1}{\ln x} \right)$  (iii)  $\lim_{x \rightarrow 1} (2-x)^{\cos \pi x}$
5. (i) Find  $\lim_{x \rightarrow 0^+} \left( e^{\frac{1}{x^2}} \right)^{x \ln(1+x)}$ ? (ii) Find  $\lim_{x \rightarrow 0} \log_{\sin x} \tan 2x$ ?
6. Evaluate:  $\lim_{x \rightarrow 0} \left( \frac{\cot x}{x} - \frac{1}{x^2} \right)$ ?
7. Evaluate:  $\lim_{x \rightarrow 0} \left( \frac{x - \sin x}{\tan^3 x} \right)$ ?
8. (i) Find  $\lim_{x \rightarrow 0} \left( \frac{x \cos x - \sin x}{x^2 \sin x} \right)$ ? (ii) Find  $\lim_{x \rightarrow 2} \left( \frac{4^x - 2^{3+x} + 16}{(x-2)^2} \right)$ ?
9. Find  $\lim_{x \rightarrow 0} \left( \frac{x^2 + 2x - 1}{x^2 - 1} \right)^{1/x}$ ?
10. Find  $\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right)^{\frac{1}{x^2}}$ ?

### Approximation, Tangent & Normal, Mean value theorem

#### Group-A (Each question carry 2 mark)

- Write the equation of the tangent to the curve  $y = |x|$  at the point  $(-2, 2)$ .
- Write the slope of tangent to the curve  $y = \sqrt{3} \sin x + \cos x$  at  $(\pi/3, 2)$ .
- What is the slope of tangent to the  $y = \sin x$  at  $x = \pi/3$ .
- What is the slope of normal to the curve  $x^{2/3} + y^{2/3} = 20$  at the point  $(8, 64)$ ?
- Write in one sentence why you can not apply Rolle's theorem to the function  $f(x) = |x|$  in  $[-1, 1]$ .
- What is the differential of  $y$  where  $x^3 y = 1/3$ .
- Write the value of  $df$  if  $f(x) = \ln(1+x)$ ,  $x=1$  and  $\delta x = 0.04$ .
- Which condition of Rolle's theorem is violated by the function  $f(x) = \sin x$  in the interval  $\left(0, \frac{3\pi}{4}\right)$ ?

#### Group-B (Each question carry 5 mark)

- State Rolle's theorem?
- State Cauchy's Mean value theorem.
- Find the equation of the tangent to the curve  $x = y^2 - 1$  at the point where slope of the normal to the curve is 2?
- Find the slope of tangent to the curve  $x = a \left( \frac{1-t^2}{1+t^2} \right)$   $y = a \left( \frac{2at}{1+t^2} \right)$  at  $t = 1/\sqrt{3}$ .

5. Find the point on the curve  $x=a(\theta-\sin\theta)$ ,  $y=a(1-\cos\theta)$  at which the tangent is parallel to the x-axis?
6. Find approximately the difference between volumes of two cubes of sides 4cm & 4.03 cm?
7. Using differential, find approximately the difference between the volume of two cubes of sides 2 cm & 2.01 cm?
8. What is acceleration at the end of 2 second of the particle that moves with the  $s=\sqrt{t}+1$ ?
9. Determine the point on the curve  $y=\ln x$  of which the tangent will be parallel to the chord joining P(1,0) and Q(e,1)?
10. Find approximate value of  $\sqrt{16.04}$  using differential?
11. Are there two points on the curve  $y^2=x$  where tangents are parallel to each other? Give reasons to your answer?
12. Give geometrical interpretation of lagrange's mean value theorem?
13. Are there two points on the curve  $y^2 = x$  where tangents are parallel to each other?

### Group-C

1. (i) Find equation of normal to  $x=\cos^3\theta$ ,  $y=\sin^3\theta$  at  $\theta=\pi/4$ .  
(ii) If the line  $y=mx+c$  touches the curve  $y^2=4ax$ , then prove that  $mc=a$
2. Show that the sum of the x-intercept and y-intercept of any tangent to the curve  $\sqrt{x}+\sqrt{y}=\sqrt{a}$  is a constant.
3. (i) Find the equation of normal to the curve  $5x^2+3y^2=23$  at (2,-1).  
(ii) Find equation of normal to the curve  $3y^2=16x$  at (3,4).
4. Prove that the length of the portion of one tangent to  $x^{2/3}+y^{2/3}=4$  intercepted between the axes is a constant.
5. Prove that curves  $ax^2+by^2=1$  &  $2x^2+3y^2=1$  intersects at right angle if  $\frac{1}{a}-\frac{1}{b}-\frac{1}{6}$
6. Prove that the sum of the cubes of intercepts on the co-ordinate axes of any tangent to the curve  $x^{3/4}+y^{3/4}=a^{3/4}$  is constant.
7. (i) Verify Rolle's theorem for the function  $f(x)=x^2+3x-10$  in  $[-5, 2]$   
(ii) Verify Rolle's theorem for the function  $f(x)=2x^2+5x-3$  on the interval  $[-3, 1/2]$
8. Using lagrange's mean value theorem prove that  $\sin x < x$  in the interval  $[0, \pi/2]$

### Increasing, Decreasing, Maxima & Minima

#### Group-A

(Each question carry 2 mark)

1. What is the value of x for which  $f(x)=3-2x^2$  is maximum?
2. Write the x coordinate of the extreme point of the function  $y=\cos x+\sin x$  at  $[0, \pi/2]$ .
3. If  $x>0$ , find the least value of  $2x+\frac{4}{x^2}$ ?
4. Write the extreme points of function  $y=x+\frac{1}{x}$ ?



- Find the interval in which the function  $\ln x/x$  is decreasing.
- Write the set of points where the function  $f(x)=x^3$  has relative extrema.
- Find the interval in which the function  $\ln x/x; x>0$  is increasing.
- What is the sub interval of  $(0, \pi)$  s.t. the function  $\sin\left(x + \frac{\pi}{4}\right)$  is increasing?
- What is the sub interval of  $(0, \pi)$  s.t. the function  $\sin\left(x + \frac{\pi}{4}\right)$  is decreasing?
- Mention the values of  $x$  for which the function  $\sin x = x^3 - 12x$  is increasing.

### Group-B

(Each question carry 5 mark)

- Determine the sub-interval of  $(-\pi/2, \pi/2)$  in which  $f(x)=\tan x - 4x$  is increasing.
- Find the open interval in which the function  $f(x)=x^{1/x}$  ( $x>0$ ) is decreasing.
- Determine the interval in which  $g(x)=\{x^3+3x+3\} / (x+1)$  is decreasing.
- Show that function  $f(x)=x^3-6x^2+15x-10$  is increasing?
- Prove that if  $f(x)=a_0+a_1x^2+a_2x^4$  &  $0<a_0<a_1<a_2$  then  $f(x)$  has only one minimum at  $x=0$ .
- Find the point on the parabola  $x^2=y$  which is nearest to the point  $(0,1)$ ?
- Answer with reasons whether the following function has a relative maximum at  $x=2$

$$\text{or not: } f(x) = \begin{cases} x & \text{if } 0 \leq x < 1 \\ 1 & \text{if } 1 \leq x \leq 2 \\ 3-x & \text{if } 2 < x \leq 3 \end{cases}$$

### Group-C

( Each question carry 6 mark)

- Show that the semivertical angle of a cone of given slant height is  $\tan^{-1} \sqrt{2}$  when its volume is maximum.
- Obtain the extreme points of  $f(x)=e^x(x^2-6x+9)$ . Ascertain whether they are maximum or minimum points. Find the extreme values at these points.
- Using the function  $f(x)=x^{1/x}; x>0$ , prove that  $e^\pi > \pi^e$ .
- Find maximum value of  $y=(1+\cos x) \sin x$  in  $[0, 3\pi/4]$
- Prove that for real values of  $x$ ,  $e \leq \frac{e^{x^2}}{x^2}$ .
- Find the extreme points & the maximum value of  $f(x)=\sin 2x + \cos 2x$ ,  $x \in [0, \pi/2]$
- (i) Find the points on the curve  $y=x^2+1$  which normal to the point  $(0,2)$ .  
(ii) Find the points of relative extrema for the function  $f(x)=x^2+3x+1$ ?
- (i) Find the altitude of right circular cylinder of maximum volume that can be inscribed in a sphere of radius  $R$ .  
(ii) Find interval s.t.  $y=\sin x - \cos x$  in  $[0, 2\pi]$  is increasing.  
(iii) Determine the interval in which the function  $f(x)=x^3-5x^2+3x+97$  is increasing & decreasing.
- (i) Find the points on the curve  $y=x^2+1$  which are nearest to the point  $(0,2)$ ? 7

(ii) Prove that the rectangle of maximum area that can be inscribed in a given circle is a square?

10. Discuss the extreme values of the function  $y = (x+2)^4(x-1)^5$ ?

### Integration

#### Group-A

(Each question carry 2 mark)

1. Write the primitive of  $\sin x + \sec x$ ?

2. Find the antiderivative of  $e^x \cdot (\tan x + \ln \sec x)$ .

3. If  $f$  is an even function and  $\int_{-2}^0 f(t) dt = 3/2$ , find  $\int_{-2}^2 f(x) dx$ ?

4. Write the value of  $\int_{-\pi/3}^{\pi/3} (x^4 \sin x^3 + x \cos x^2) dx$ ?

5. Find  $\int_{-\pi/2}^{\pi/2} \sin^5 x \cos (nx/2) dx$ ?

6. Find  $\int_{-2}^{-1} |-x| dx$ ?

7. Find  $\int_{-\pi}^{\pi} x^5 \cos x dx$ ?

8. Find  $\int_{-1}^0 [x] dx$ ?

9. Find  $\int_1^3 \tan^{-1} x dx + \int_1^3 \cot^{-1} x dx$ ?

10. Find  $\int_{-\pi/2}^{\pi/2} \sin^5 x dx$ ?

11. Find  $\int_{-\pi/2}^{\pi/2} \sin^3 x dx$ ?

12. Find  $\int \log x dx$ ?

13. Find  $\int_0^2 |x-1| dx$ ?

14. Find  $\int e^{x^2} 2x dx$

15. Find  $\int |x| dx$ ;  $x < 0$ ?

16. Find  $\int \log e^x dx$ ?

17. Find  $\int_0^{1/2} 2y dy + \int_{1/2}^1 2y dy$ ?

18. Find  $\int_0^{\pi/4} \tan x dx$ ?

19. Find  $\int e^x \cos x dx + \int e^x \sin x dx$ ?

20. Find  $\int \frac{1-t}{1+t^2-1} dt$ ?

21. Find  $\int \frac{1+\frac{1}{x^2}}{x-\frac{1}{x}+4} dx$ ?

22. Find  $\int \frac{3dx}{(x-1)(x+2)}$ ?

23. Find  $\int \left( \sqrt{a^2 - x^2} + \frac{x^2}{\sqrt{a^2 - x^2}} \right) dx$ ?

24. Find  $\int 2 \sin(\alpha - \beta)x \sin(\alpha + \beta)x dx$ ?

25. Find  $\int \frac{dx}{5-4x+x^2}$ ?

26. Find  $\int 2e^{\ln x} dx$

27. Find  $\int \frac{\cot^2 x \cdot \operatorname{cosec}^2 x}{x^2} dx$ ?

28. Find  $\int_{-1}^1 ([x] + 3) dx$ ?



29. Find  $\int \tan^2 x \cdot x - \int \sec^2 x dx$ ?
30. Find  $\int \sin^2 x d(\sin x)$ ?
31. If  $\int_a^b f(x) dx = 1$  then find  $\int_b^a kf(t) dt$ ?
32. Write true or false :  $\int_a^b x dy = \int_a^b y dx$ ?
33. Evaluate  $\int_0^2 |3-2x| dx$ ?
34. Find  $\int 2^x dx$ ?
35. If  $F(x) = \int_0^x e^{2t} \sin 3t dt$ , find  $F'(x)$ ?
36. If  $\int_0^1 f(t) dt = 2$  &  $\int_2^1 f(u) du = -1$  then find  $\int_0^2 f(x) dx$ ?
37. Find  $\int 2x \operatorname{cosec}^2 x^2 dx$ ?
38. Find  $\frac{d}{dx} \int_{250}^{300} (x^4 + 5x^3)^2 dx$ ?
39. Find m s.t.  $\int x^m dx = \frac{x^{m+1}}{m+1}$ ?
40. Find  $\int_e^{\ln(\operatorname{cosec}^2 x)} dx = ?$
41. Find  $\int_0^1 \sin^2 x dx + \int_0^1 \cos^2 x dx - \int_0^1 dt$ ?
42. Find  $\int \frac{dx}{1+e^{-x}}$ ?
43. Find  $\int_0^1 |x-1| dx$ ?
44. If  $\int_a^b f(x) dx = k$ , find  $-\int_b^a f(x) dx$
45.  $\int \cos \frac{\pi}{4} dx$
46. Find  $\int_{-1}^1 |x| dx$
47. Find  $\int_{-1}^1 e^x dx$ ?
48. Find  $\int_0^2 |x-3| dx$ ?
49. Find  $\int_0^1 |x+1| dx$ ?
50. Find  $\int \sin \frac{x}{2} \cdot \cos \frac{x}{2} dx$
51. Find  $\int_1^3 [x] dx$ ?
52. If  $\int_0^1 f(1-x) dx = 2$ , find  $\int_0^{1/2} f(2t) dt$ ?
53. Find  $\int e^x \{f(x) + f'(x)\} dx$ ?
54. Find  $\int 2^x 4^{-x} \cdot 2 dx$ ?
55. Find  $\int_a^{\infty} dx$ . Find a?
56. What is the antiderivative of  $2^{2^x+x}$ ?
57. 'Every function has an antiderivative' is it true?
58. What is the value of  $\int \frac{d}{dx} f(x) dx - \frac{d}{dx} (\int f(x) dx)$
59. If  $\int_1^2 f(x) dx = \lambda$  then what is the value of  $\int_1^2 f(3-x) dx$ ?
60. What is the value of  $\int_{-1}^1 \frac{dx}{1+x^2}$ ?
61. What is  $f'(x)$  if  $f(x) = \int_0^x e^{2t} \sin 3t dt$ ?

62. Write the value of  $u \int v dx - \int u' \left( \int v dx \right) dx - v \int u dx + \int v' \left( \int u dx \right) dx$  ?

63. Find  $\int e^{\ln(\csc^2 x - \cot^2 x)} dx$  ?

64. Find  $\int \frac{dx}{\cos^2 x \sin^2 x}$  ?

65.  $\int \frac{\cot x dx}{\ln \sin x}$  ?

### Group-B

(Each question carry 5 mark)

1. Integrate  $\int \frac{dx}{\sqrt{2x-x^2}}$  ?

3. Find  $\int \cot^{-1} x dx$  ?

5. Evaluate  $\int_0^1 \left[ \frac{x}{3} \right] dx$  ?

7.  $\int e^x (\cot x + \ln \sin x) dx$  ?

9. Evaluate :  $\int_0^1 [3x] dx$  ?

11.  $\int \frac{dx}{\sqrt{5+4x-x^2}}$  ?

13.  $\int \frac{e^x \sin x}{\sqrt{16 + \cos^2 e^x}} dx$  ?

15.  $\int \frac{dx}{\sqrt{16-6x-x^2}}$  ?

17.  $\int_0^{2\pi} x \csc x dx$  ?

19.  $\int \frac{dx}{2 - \sin x}$  ?

21.  $\int_0^1 x^6 (1+x^7) dx$  ?

23.  $\int \frac{\sin x}{\sin(x-2)} dx$  ?

25.  $\int \tan^{-1} x dx$  ?

27.  $\int_0^{\pi/2} \frac{x^7 \cos x}{1 + \sin^2 x} dx$  ?

29.  $\int_0^{\pi/2} \log \tan x dx$  ?

2. Find  $\int \frac{\cos x}{\sin^2 x + \sin x} dx$  ?

4. Find  $\int_0^{\pi/2} \frac{5 \sin^3 x}{\sin^3 x + \cos^3 x} dx$  ?

6. Find  $\int \frac{\cos 3x \cdot \cos x}{1 + \cos 2x} dx$  ?

8.  $\int_0^{\pi/4} \frac{dx}{\cos x (\cos x + \sin x)}$  ?

10. Evaluate ?  $\int_0^4 |8-3x| dx$  ?

12. Find  $\int \frac{(4x-5)dx}{x^2-x-2}$  ?

14. Find  $\int x^2 e^{x^3} \sin x^3 dx$  ?

16.  $\int \frac{3x dx}{(x-4)(x+2)}$  ?

18.  $\int_0^3 x^2 e^{x^3} dx$  ?

20.  $\int \frac{x^3 dx}{x^4 - x^2 - 2}$  ?

22.  $\int_1^3 x \log x dx$  ?

24.  $\int \frac{dx}{(x^2 + 3x + 2)^2}$  ?

26.  $\int \frac{dx}{3x^2 - 3x + 2}$  ?

28.  $\int \frac{x dx}{4x^2 + 5}$  ?

30.  $\int \sqrt{2-x^2} dx$  ?



$$31. \int \sqrt{2x} \sqrt{x^2} dx?$$

$$33. \int \frac{2}{-12x+5} dx?$$

$$35. \int 5^{\cos^2 x} \sin 2x dx?$$

$$37. \int_0^4 [\sqrt{x}] dx$$

$$32. \int_0^4 \frac{2dx}{\sqrt{x^2+9}}?$$

$$34. \int \sin^4 x \cdot \cos^3 x dx$$

$$36. \int \frac{\sec x \cdot \operatorname{cosec} x}{\ln \tan x} dx$$

$$38. \int \frac{e^x - 1}{e^x + 1} dx$$

### Group-C

(Each question carry 10 mark)

$$1. \int \frac{x^2+1}{x^2-10x+29} dx?$$

$$3. \int \frac{dx}{\cos x (1+2\sin x)}?$$

$$5. \int \frac{1}{2\sin x + \cos x + 3} dx?$$

$$7. \int_0^{\pi/2} \frac{\sin x (7 - \cos x)}{(1 - \cos^2 x)(2 - \cos x)} dx?$$

$$9. \int x^2 (\sin^4 x + \cos^4 x) dx?$$

$$11. \int_0^{\pi/2} \frac{(1 - \sin^2 x)(4\sin x - 3)}{(4 - \cos^2 x)(\sin x + 4)\cos x} dx?$$

$$13. \int_0^{2\pi} x |\sin x| dx?$$

$$15. \int_0^{\pi} \frac{dx}{1 + \sin x}?$$

$$17. \int \frac{2\sin x - 3\cos x}{4\sin x + 3\cos x} dx?$$

$$19. \int \frac{3x+4}{\sqrt{2x-3}} dx?$$

$$21. \text{Find antiderivative of } \sqrt{\frac{1}{4+x^2}}?$$

$$23. \int \frac{x^3}{x^4 - x^2 - 2} dx?$$

$$25. \int \sqrt{3+2x-x^2} dx?$$

$$27. \int \frac{dx}{\sqrt{5x-6x^2}}?$$

$$2. \int_0^1 \frac{\ln(1+x)}{1+x^2} dx?$$

$$4. \int x^3 \cos^2 x dx?$$

$$6. \int_0^{\pi/2} \frac{1}{4+5\sin x} dx?$$

$$8. \int \frac{\ln x dx}{x^2}?$$

$$10. \int \frac{\cos x dx}{\sin^4 x}?$$

$$12. \text{Determine : } \int \frac{dx}{(x-2)\sqrt{3x^2-16x+24}} dx?$$

$$14. \int_0^{\pi} \frac{x dx}{1 + \sin x}?$$

$$16. \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx?$$

$$18. \int \frac{1}{(2x+5)\sqrt{x+2}} dx?$$

$$20. \int \frac{dx}{5 + \sin x}?$$

$$22. \int \frac{2x+5}{(x+2)^{7/2}} dx?$$

$$24. \int \frac{dx}{2 - \sin x}?$$

$$26. \int (x^2 - 2x + 7)\sqrt{x+1} dx?$$

$$28. \int \frac{11x+6}{10x^2 + 11x+3} dx?$$

$$29. \int_0^{\pi/2} \sqrt{\tan \theta} d\theta ?$$

$$31. \int \frac{x^2 dx}{(x-1)^2(x-2)}$$

$$33. \int \frac{dx}{(1+x)\sqrt{1-x^2}}$$

$$35. \int \frac{\cos x}{\sin 2x + \sin x} dx$$

$$37. \text{ Prove } \int_0^{\pi/4} \cos^2 2x \cdot \sin^3 4x dx \text{ is } 1/6 ?$$

$$30. \int_0^{\pi} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx ?$$

$$32. \int_0^{\pi/2} \frac{\cos x dx}{\sin x + \cos x}$$

$$34. \int e^{-3x} \sin 4x dx$$

$$36. \int_0^{\pi/4} \cos^2 2x \cdot \sin^3 4x dx$$

### Application of integration (Area and Differential equations)

**Group-A** (Each question carry 2 mark)

1. Write the order & degree of the differential equation  $\left(\frac{dy}{dx}\right)^8 + \left(\frac{d^2y}{dx^2}\right)^2 = 0 ?$

2. Write the degree of the differential equation  $\left(3x + 3y \frac{d^3y}{dx^3}\right)^{2/3} = \frac{dy}{dx} + \frac{d^2y}{dx^2} ?$

3. Write the degree of the differential equation  $\left(\frac{dy}{dx}\right)^4 + y^5 = \left(\frac{d^3y}{dx^3}\right) ?$

4. Find the degree and order of  $\left[\frac{1 + \left(\frac{dy}{dx}\right)^2}{\left(\frac{d^2y}{dx^2}\right)}\right]^{3/2} = a ?$

5. Find the degree and order of  $\sqrt{\frac{dy}{dx} + 1} + \frac{d^2y}{dx^2} = 0 ?$

6. Find order and degree of  $\frac{d^3y}{dx^3} = \left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^4 + y ?$

7. Find the particular solution of  $\frac{dy}{dx} = (1+x)^4$ ;  $y=0$  when  $x=-1$ ?

8. Given general solution as  $y=(x^2+c)e^{-x}$  of a differential equation. What is particular solution if  $y=0$  when  $x=1$ ?

9. find order of the differential equation whose general solution is  $y=ax^2+b$ ,  $a$  &  $b$  being arbitrary constants?

10. Write the differential equation whose solution is  $y=e^{x+a}$ ?  
 11. Write differential equation of the parabola  $y^2=4x+12$ ?  
 12. How many arbitrary constants does general solution of the differential equation

$$\frac{d^2y}{dx^2} = \sin x + \cos x \text{ contain?}$$

13. What is the differential equation whose general solution is  $y=mx$  where  $m$  is an arbitrary constants?  
 14. Write the order & degree of the differential equation whose general solution is  $\sqrt{x} = A\sqrt{y} + 1$ ?

15. Form the differential equation whose solution is  $y=ax^2+bx$ ?

16. Eliminate the arbitrary constant  $C$  from  $y^2=2Cx$ ?

17. Write the number of constants in the solution of the differential equation

$$\frac{d^2y}{dx^2} = x^3 \cos x - 8x^3 + 2^3 + 2^3. (x=0 \Rightarrow \frac{dy}{dx} = 1)?$$

18. Write the differential equation of which  $y=ce^x+ce^{-x}$  is the general solution?

19. Write the differential equation whose general solution is  $y=3x+k$ ?

20. Write the solution of  $dy/dx=8x$ , given  $y=2$ ?

21. Write the particular solution of  $\frac{dy}{dx} = \frac{1}{1+x^2}, y(0)=1$ ?

22. Write the solution of  $\sqrt{4 + \frac{dy}{dx}} = 2$ ?

23. Write an integrating factor of  $dy/dx+x=0$ ?

24. Write the linear form of the differential equation  $\frac{dy}{dx} - \frac{1}{x}y = -y^2$ ?

25. Write the solution of  $\frac{d^2y}{dx^2} = 0$ ?

26. Write the solution of  $\frac{d^2y}{dt^2} = 0$ ?

27. If the homogeneous form of the differential equation

$$\frac{dy}{dx} = \frac{x+y+1}{x-y+1} \text{ is } \frac{dY}{dX} = \frac{X+Y}{X-Y}, \text{ then what is the relation between } Y \text{ \& } y?$$

28. Find the particular solution of  $\frac{dy}{dx} = \sin x$ ;  $y=2$  when  $x = \pi$ ?

29. What is the area bounded by  $x=e^y, x=0, y=0, y=1$ ?

30. Write the area bounded by  $y=-2x, y=0, x=1, x=3$ ?

31. Write the area bounded by  $y=e^x, x\text{-axis}, x=0 \text{ \& } x=2$ ?

32. Write the formula in integral form for finding the area bounded by the curve  $y^2=x$  \&



the lines  $y=0$ ,  $x=a^2$ ,  $x=b^2$  ( $a^2 < b^2$ )?

**Group-B** (Each question carry 3 mark)

1. Find the integrating factor of the solution of the differential equation :

$$(x - \ln y) \frac{dy}{dx} = -y \cdot \ln y$$

2. Find the differential equation whose primitive is  $y = a \cos x + b \sin x$

3. Write the order & degree of the differential equation

$$\left( \frac{d^2 y}{dx^2} + \frac{dy}{dx} \right)^5 + \left( \frac{d^3 y}{dx^3} \right)^2 = x^4 \sqrt{3 \frac{d^3 y}{dx^3} + 1}$$

4. Obtain the differential equation whose primitive is  $y = Ae^{2x} + Be^{2x}$ ?

5. Find the factor that should be multiplied with the differential equation

$$\cos x \frac{dy}{dx} + y \cdot \sin x = 3 \text{ to make it integrable?}$$

6. Find the differential equation whose general solution is  $ax^2 + by^2 = c$ ?

7. Find the solution of  $\frac{d^2 y}{dx^2} = e^x + 1$ , given that  $y=2$  &  $\frac{dy}{dx}=4$  when  $x=0$ ?

8. Find the general solution of  $\frac{d^2 y}{dx^2} = e^{2x} + e^{-x}$ ?

9. Solve :  $x^2 \sqrt{y^2 - 3} dx + y \sqrt{x^3 + 3} dy = 0$ ?

10. Solve the following differential equation  $x \sqrt{y^3 + 5} dx + y^2 \sqrt{x^2 + 3} dy = 0$ ?

11. Solve :  $\frac{d^2 y}{dx^2} + 40x^3 + 12x^2 + 8 = 0$ ?

12. Solve  $\frac{dy}{dx} = x^5 \cdot \ln x$ ?

13. Solve :  $y dx - x dy = xy dx$ ?

14. Find the integrating factor of the solution of the differential equation :

$$(1 + y^2) dx + x dy = \tan^{-1} y dy$$

15. Write the order & degree of the differential equation  $\frac{d^2 y}{dx^2} = \frac{2y^3 + \left( \frac{dy}{dx} \right)^4}{\sqrt{\frac{d^2 y}{dx^2}}}$ ?

16. Reduce  $x \frac{dy}{dx} + y = xy^2$  into a linear differential equation?

17. Find the integrating factor of the solution of the differential equation :

$$(x + \tan y) dy = \tan y dx$$

18. Find the particular solution of  $\frac{dy}{dx} + \frac{1 + y^2}{1 + x^2} = 0$ ,  $y(-1) = \sqrt{3}$ ?

19. Solve  $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$  ?

20. Find the area of the trapezium bounded by the sides  $y=x$ ,  $x=0$ ,  $y=3$ ,  $y=4$ .

21. Determine the area of the region bounded by the curve  $y^4=x^3$  and the double ordinate through  $(2,0)$ ?

22. Find the area bounded by  $y=e^x$ ,  $y=0$ ,  $x=4$ ,  $x=2$ ?

23. Find area of parabola  $y^2=4ax$  about its latus rectum?

24. Find the area enclosed by the curve  $y^2 = x$  and the straight lines  $x=0$ ,  $y = 1$ ?

25. Find by integration the area bounded by the straight lines  $y=x$ ,  $y=0$ ,  $x+2y=3$ ?

**Group-C** ( Each question carry 6 mark)

1. Solve the following differential equation :  $x \frac{dy}{dx} + y = y^2 \ln x$  ?

2. Solve :  $\frac{dy}{dx} - y \cot x = xy^4$  ?

3. Solve :  $\frac{dy}{dx} = (x+y)^2$  ?

4. Solve :  $\frac{d^2y}{dx^2} = \frac{1}{x(x+1)} + \operatorname{cosec}^2 x$  ?

5. Solve :  $\frac{dy}{dx} = \frac{y^2 + xy}{x^2 - xy}$ , given that  $y(1)=1$ ?

6. Solve :  $\frac{d^2y}{dx^2} = 4e^{2x} + \cos x + \sec^2 x$ , given that  $y(0)=2$ ,  $y'(0)=4$ ?

7. Solve :  $\frac{dy}{dx} = \frac{y-x+1}{y+x+5}$  ?

8. Find the general solution of  $e^{-3x} \frac{d^2y}{dx^2} = 3x$ ?

9. Find the general solution of  $\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{x^2}$  ?

10. Find the general solution of  $y \frac{dx}{dy} + x = x^2 \log y$  ?

11. Solve :  $(x+y+1) \frac{dy}{dx} = 3$  ?

12. Solve :  $\frac{dy}{dx} = \frac{y-x+1}{y+x+5}$  ?

13. Solve :  $\operatorname{cosec} x \frac{d^2y}{dx^2} = x$  ?

14. Solve :  $\sec x \frac{d^2y}{dx^2} = x$  ?

15. Find the particular solution of :  $\frac{dy}{dx} = \frac{x \ln x}{3y^2 + 4y}$ ,  $y(0)=4$ ?

16. Find the particular solution of :  $e^{-x} \frac{d^2y}{dx^2} = x^2$ ,  $y=3$ ,  $\frac{dy}{dx} = 2$  when  $x=0$ ?

17. Solve :  $(x+y)dy + (x-y)dx = 0$ ?

18. Solve :  $x \log x \frac{dy}{dx} + y = 2 \log x$  ?



19. Solve :  $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ ,  $x=1$ ,  $y=\sqrt{3}$  ?
20. Solve :  $\frac{d^2y}{dx^2} = 6x$ ,  $y=1$ ,  $\frac{dy}{dx} = 2$  when  $x=0$ ?
21. Form the differential equation whose general solution is  $y=ae^x+bx$ ?
22. Solve the differential equation  $(x+2) dy + tany dx=0$ ?
23. Solve  $dy/dx=e^{x-y}$  ?
24. Solve :  $\frac{d^2y}{dx^2}=3x^2-x+1$ ;  $y(0), y'(0)=1$ ?
25. Find the area of the region bounded by  $y=\cos x$  & the part of the  $x$ -axis between  $x=-\pi/2$  and  $x=3\pi/2$  ?
26. Solve :  $\frac{d^2y}{dx^2} = x e^x$ ,  $y(0)=1$ ,  $y(1)=0$  ?
27. Solve :  $x \frac{dy}{dx} + y = y^2$  ?
28. Solve :  $\frac{dy}{dx} = \frac{xy}{1+x^2}$  ?
29. Solve :  $\frac{dy}{dx} + \frac{e^y}{1+x} = \frac{e^x}{1+x}$  ?
30. Solve :  $\frac{d^2y}{dx^2} = \sin x \cdot \cos 3x$ ?
31. Find the particular solution of  $\log \frac{d^2y}{dx^2} + x=0$ , when  $x=0$ ,  $y=0$ ,  $dy/dx=0$ ?
32. Solve :  $\frac{d^2y}{dx^2} = \sin 3x$  ?
33. Solve  $e^x \frac{d^2y}{dx^2} = x$ ?
34. Solve  $(2x+y+1)dx + (4x+2y-1)dy=0$ ?
35. Show that the area bounded by the parabolas  $y^2=4x$  &  $x^2=4y$  is equal to the area bounded by the curve  $x^2=4y$  and the lines  $y=0$ ,  $x=4$ ?
36. Determine the area included between the parabola  $y^2=x$  & the circle  $x^2+y^2=2x$ ?
37. Find the area of portion of ellipse  $x^2/12+y^2/16=1$  bounded by major axis and the double ordinate  $x=3$ .
38. Find the area of the region bounded by the parabola  $y^2=x$  and the straight line  $y=2x$  ?
39. Find the area of the region bounded by the parabola  $x^2=4ay$  and the straight line  $y=a$ ?
40. Find the area of the region bounded by the parabola  $y=x^2$  and  $y=0$ ,  $x=1$  ?
41. Find the area of the region enclosed between the line  $y=4x-1$  & the parabola  $y^2=2x$  ?

### Permutation, Combination

#### Group-A

( Each question carry 2 mark)

- Find the value of  $n$  if  $C(n, 20) = C(n, 23)$ ?
- $C(20, r) = C(20, r-10)$  find  $r$  ?
- If  $P(8, r)=56$  find  $r$  ?



4. What is  $r$  if  $C(6, r) - C(5, 3) = C(5, 2)$ ?
5. Is  $C(n, r)$  divisible by  $r!$  &  $(n-r)!$ ?
6.  $n! / (n-2)!$  is satisfied by what value of  $n$ ?
7. If  $C(n, r)$  is odd or even number?
8. If  ${}^nC_r = {}^nC_s$  find  $r$ ?
9. If  ${}^nP_2 > 131$ , then what is the minimum value of  $n$ ?
10. What is the number of ways in which 5 different books can be arranged on a shelf?
11. Is the number of ways in which 5 persons can sit on a bench same as when they sit on a round table?
12. If 7 points out of 12 are in the same straight line, then how many triangles are formed?
13. How many hexadecimal two digit positive integers are there?
14.  $P(n, 1) = 1005$  find  $n$ ?
15. In how many ways can 10 people line up at a ticket window of a cinema hall?
16. If  $|x| = 1, |y| = 2, |z| = 3$ , then how many points in  $R^3$  are there having co-ordinates  $(x, y, z)$ ?
17. Mention the number of ways in which 15 different books can be divided equally among 3 boys?
18. A man has 5 daughters. he wants to visit a planetarium alone or taking one or more of the daughters. In how many ways his wish can be fulfilled?
19. How many even numbers consists of 4 digits can be formed by using the digits 1, 2, 3, 4, 5, 7?
20. How many two digit numbers can be formed with digits 2, 5, 7, 8, 9, 0 repetition of digits being allowed?
21. What is  $r$  if  ${}^{16}P(15, r) = {}^{13}P(16, r)$ ?

**Group-B** (Each question carry 5 mark)

1. Find the number of ways that 5 examination papers be arranged so that the best and the worst papers do not come together?
2. A committee of 6 is to be formed from 7 gentlemen & 4 ladies. In how many ways can this be done so as to include at least two ladies?
3. In how many ways can eight letters be posted in two of letter boxes?
4. Prove that  $P(2n, n) = 2n [1.3.5... (2n-1)]$ ?
5. How many four digits numbers can be formed with the digits 4, 3, 2, 0 digits not being repeated?
6. Find the number of diagonals of a polygon of  $n$  sides?
7. Find  $n$  &  $r$  if  ${}^nP_r = 1680$  &  ${}^nC_r = 70$ ?
8. How many numbers each lying between 400 & 1000 can be formed with the digits 2, 3, 4, 5, 6, 0?
9. Find number of triangles that can be formed by joining the angular points of a decagon?

10. If the number of combinations of  $2n$  different things taken one or two or three at a time be in AP. Find  $n$ ?
11. Find  $n$  if  ${}^{18}C_3 = 5 {}^nC_5$  ?
12. How many natural numbers less than 1000 can be formed using 0, 2, 3, 5 and no digit is repeated?
13. How many different numbers of six digits (without repetition) can be formed from the digits 3, 1, 7, 0, 5, 9 & how many of them are divisible by 5 ?
14. Prove that only 18 numbers can be formed by using all the digits 1, 2, 3, 4, 3, 2, 1 so that odd digits always occupy the odd places?
15. A number is chosen at random from integers 1, 2, 3, 4, 5, 6 ...., 20. What is the probability that it is a multiple of 3 or 5 ?
16. How many different numbers of six digits (without repetition) can be formed from the digits 3, 1, 7, 0, 5, 9 & how many of them are divisible by 5 ?

#### Group - C

( Each question carry 10 mark)

20. In how many ways can 4 men & 4 women sit at a round table so that no two men can occupy adjacent places?
21. In how many ways can a student choose 5 courses out of the courses  $C_1, C_2, \dots, C_9$  if  $C_1, C_2$  are compulsory &  $C_6, C_8$  can not be taken together?
22. In how many ways can 4 persons be selected from 10 persons so as to :  
(i) include always a particular person?  
(ii) exclude always two particular persons?
23. Show that  $P(2n, n) = 2^n \{1.3.5... (2n-1)\} = (n+1)(n+2) \dots 2n$ ?
24. A cricket team having 11 players is to be chosen from 8 batsmen & 5 bowlers. Find in how many ways can the team be chosen so as to include at least 3 bowlers?
25. If  $C(2n+1, n-3) : C(2n-1, n-4) = 55/9$ , find the value of  $n$ ?
26. Out of 8 sailors on a boat, 3 can work only on one side and 2 only on the other side. In how many ways can the sailors be arranged on the boat?
27. Prove that :  ${}^{4n}C_{2n} : {}^{2n}C_n = \{1.3.5...(4n-1)\} : [1.3.5...(2n-1)]^2 2^n$  ?
28. Prove that  $C(n+1, r) + C(n, r-1) + C(n, r-2) = C(n+2, r)$  ?
29. Establish the formula :  ${}^nP_r = n! / (n-r)!$  ?
30. Find number of ways in which an arrangement of four letters can be made from the letters of the word "PROPORTION"
31. There are four boys and four girls. In how many ways can they be seated in a row so that all the girls do not sit together?

#### Binomial Theorem

#### Group - A

( Each question carry 2 mark)

1. What is the number of terms in the expansion of  $(1+x)^7 (1-x)^7$  ?
2. How many middle terms are there in the expansion of  $(1+2x+x^2)^{13}$  ?
3. Write the co-efficient of  $x^2$  in expansion of  $(x-2/x)^{10}$  ?
4. If  $2(1+x)^n = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ , then find the value of  $a_0 + a_1 + a_2 + \dots + a_n$  ?



- What is the middle term in the expansion of  $(a/b + b/a)^6$ ?
- If  $m_1$  &  $m_2$  denote co-efficient of two middle terms in  $(1+x)^{2n+1}$  then which of the following is true?  $M_1=M_2$ ,  $M_1<M_2$ ,  $M_1>M_2$ ?
- Are the co-efficient of the middle term in the expansion of  $(1-x)^{21}$  &  $(x-1)^{21}$  equal?
- If  $(1+x)^n = \sum_{i=0}^n C_i x^i$ , then find  $\sum_{i=0}^{100} C_{2i}$ ?
- Find  ${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \dots + {}^{2n+1}C_{2n+1}$ ?
- Write the value of  ${}^9C_1 + {}^9C_2 + {}^9C_4 + {}^9C_6 + {}^9C_8$ ?
- What is the middle term in the expansion of  $\left(\frac{1}{2}a + \frac{1}{3}b\right)^8$ ?

**Group - B** (Each question carry 5 mark)

- Determine the term independent of  $x$  in the following expansion of  $(x+1/x)^{2n}$ ?
- Find the co-efficient of  $x^{15}$  in the expansion of  $(x-x^2)^{10}$ ?
- Find the term independent of  $x$  in the expansion of  $(x^3-1/x^{13})^{13}$ ?
- In the expansion of  $(a-1/a)^{11}$ , find the co-efficient of the term involving  $1/a$ ?
- Write the expansion of  $(x+1)^6$  using pascal's triangle?
- If  $a = 0.2$ ,  $b = 1.12$  then find  $(a+b)^5 - 5(a+b)^4b + 10(a+b)^3b^2 - 10(a+b)^2b^3 + 5(a+b)b^4 - b^5$ ?
- Find  $n$  and  $x$  such that  $(1+x)^n = 1 - \frac{3}{4} + \frac{(3)(5)}{(4)(8)} - \frac{(3)(5)(7)}{(4)(8)(12)} + \dots$

**Group - C** (Each question carry 10 mark)

- Find the term independent of  $x$  in the expansion of  $(1+4x^2)(x^2+1/x)^{11}$ ?
- Prove that:  $C_0 + 3C_1 + 5C_2 + \dots + (2n+1)C_n = (n+1)2^n$ ?
- If  $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_r x^r + \dots + C_n x^n$ . Prove that  $\frac{C_0}{1} + \frac{C_2}{3} + \frac{C_4}{5} + \frac{C_6}{7} + \dots = \frac{2^n}{n+1}$
- (a) Prove:  $1 + \frac{C_1}{2} + \frac{C_2}{3} + \dots + \frac{C_n}{n} + \dots = \frac{2^{n+1}-1}{n+1}$ ? (b) Prove:  ${}^nC_r = \frac{n!}{r!(n-r)!}$ ?
- If  $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_r x^r + \dots + C_n x^n$ . Prove that  $C_0 + 3C_1 + 5C_2 + \dots + (2n+1)C_n = (n+1)2^n$ ?
- Prove that  $(C_0)^2 + (C_1)^2 + \dots + (C_n)^2 = 2n!/(n!)^2$ ?
- Prove:  $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \dots + \frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$ ?
- Prove that:  $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + \dots + n\frac{C_n}{C_{n-1}} = \frac{n(n+1)}{2}$ ?
- Prove:  $\frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + \dots = \frac{2^n-1}{n+1}$ ?



10. Find the term independent of  $x$  in the expansion of  $(x^2 - 2/x^3)^{15}$  ?

11. Find the term independent of  $x$  in the expansion of  $\left(\frac{1}{2} - 10x + 6x^2\right)\left(\frac{1}{x} - x\right)^{10}$  ?

12. Prove  ${}^nC_0 - \frac{{}^nC_1}{2n} + \frac{{}^nC_2}{3n^2} - \frac{{}^nC_3}{4n^3} \dots + (-1)^n \frac{{}^nC_n}{(n+1)n^n} = \frac{n^{n+1} - (n-1)^{n+1}}{(n+1)n^n}$  ?

13. Prove that  ${}^nC_0 = 2 \cdot {}^nC_1 + 3 \cdot {}^nC_2 + \dots + (n+1) \cdot {}^nC_n = 2^n + n \cdot 2^{n-1}$  ?

### Probability.

**Group - A** ( Each question carry 1 mark)

1. Three dice are rolled. What is the probability that the same numbers will appear on all the dice ?
2. Find probability that two cards arbitrarily drawn from a pack of 52 cards will both be aces?
3. What is the probability of impossible event?
4. What is the probability of getting a black card from a pack of 52 cards?
5. What is the probability of getting four kings from a pack of 52 cards?
6. In a simultaneous toss of two coins what is the probability of getting exactly one tail?
7. In a single throw of a dice find the probability of getting an odd number?
8. If a coin is tossed thrice, is  $\{(h, h)\}$  an event?
9. If four coins are tossed what is the probability of getting at least one head?
10. What is the probability that one digit positive integer is even?
11. Define independent events?
12. A couple have two daughters. What is the probability that their next child will be a daughter?
13. Fill the blank :  $P(A) + P(A^c) = \dots$  ?
14. Write the probability of getting exactly two heads in a single throw of two coins?
15. If  $P(A) = 1/2$  and  $P(A \cap B) = 1/3$  then what is the probability of  $(A - B)^c$  ?
16. A bag contains 7 white and 9 black balls. If a ball is drawn at random, What is the probability that it is white?

**Group-B** ( Each question carry 3 mark)

1. If two dice are tossed once find the probability of getting a score less than 10 ?
2. Find the probability of having at least one head in 5 throws of a coin?
3. If  $A$  &  $B$  are two events such that  $P(A) = 0.6$ ,  $P(B) = 0.5$  &  $P(A \cap B) = 0.2$ , find  $P(A/B^c)$ ?
4. If  $A \subseteq B$ , Prove that  $P(A) \leq P(B)$  ?
5. If  $A$  and  $B$  are two events with  $P(A) = 3/8$  and  $P(A \cap B) = 1/4$ , find  $P(A \cap B^c)$ ?
6. Two dice are rolled in succession. Find the probability that the first dice shows at most 3 & the second dice shows an odd number not less than 3?
7. If a card is selected at random from a deck of 100 cards numbered from 6 to 105,



- find the probability that the number on the card is divisible by either 2 or 5 ?
8. In a single throw of two dice determine the probability of getting a total of 7 or 8 ?
  9. A coin is tossed three times. Find the probability that head & tail will show alternatively?
  10. What is the probability that a leap year selected at random will contain 53 sundays?
  11. In a single throw of two dice find the probability of getting a total of 11?
  12. A pair of dice is rolled once. Find the probability of a total of at most 9 ?
  13. Prove  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$  ?
  14. A box contains 25 tickets numbered 1 to 25. Two tickets are drawn at random. What is the probability that the product of numbers is even?
  15. Find the probability of getting six different faces in a throw of six different dice?
  16. A five digit number is formed by the digits 1, 2, 3, 4, 5 without repetition. Find the probability that the number so formed is a multiple of 4?
  17. What is the probability of a binary two digit positive integer odd?
  18. If A & B are two events such that  $P(A) = 3/8$ ,  $P(B) = 1/2$  &  $P(A \cap B) = 1/4$ , find  $P(A/B^c)$ ?
  19. A student is given 10 true-false questions in exam. If he gets 8 or more correct answers he passes the exam. Give that he guesses at the answer to each question, compute the probability that he passes the examination?
  20. A number is chosen at random from integers 1, 2, 3, 4, 5, 6, ..., 20. What is the probability that it is a multiple of 3 or 5 ?

### Group - C

( Each question carry 10 mark)

1. A bag contains 4 white, 3 black & 2 red balls; a second bag contains 5 blue, 3 white & 4 black balls; a third has 2 green, 3 red & 5 white balls. A bag is selected at random and ball is drawn from it. Find the probability that the ball is white if the probability of choosing the first bag is three times that of choosing the second and four times that of choosing the third bag?
2. (a) If two events A & B are independent then prove that  $A^c$  & B are independent?  
(b) Find the probability that in a well shuffled pack of 52 cards, the four kings will remain together & also the four queens will be together?
3. Three N.C.C. cadets A, B & C took part in a shooting competition. Their probabilities of hitting the target are respectively 0.8, 0.9, 0.7. They fire once each. What is the probability that at least two shots hit the target?
4. Find the probability of obtaining a total of 9 in a single throw of two dice?
5. From a bag containing 3 black and four white balls two balls are drawn at random one after another. Find the probability that the second ball selected is white?
6. From a pack of cards containing 5 black & 4 red cards, two cards are drawn one after other. Find the probability that first card drawn is black if the second card is known to be red?
7. Four balls are drawn successively (and not replaced) from a bag containing 6 white and 4 black balls. Find the probability that they are of different colours?



8. State the axioms of probability and prove the following :
- $A \subset B \Rightarrow P(A) \leq P(B)$
  - $P(A-B) = P(A) - P(A \cap B)$
9. A bag contains 8 white & 6 red balls. If 5 balls are drawn at random find the probability that 3 are white balls?
10. A & B play a game alternatively throwing a pair of dice. One who throw 8 first wins the game. A starts the game and the game continues for three throws. Find the probability that A wins in the last throw?
11. A die is thrown 5 times, what is the probability of getting exactly 3 fours?
12. If A & B are independent events prove that :  
 $P(A' \cap B') = P(A')P(B')$  ?
13. Prove that  $P(A \cap B) \geq P(A) + P(B) - 1$  ?
14. If  $A_1, A_2, \dots, A_n$  are events then prove that :  $P(A_1 \cup A_2 \cup \dots \cup A_n) \leq P(A_1) + P(A_2) + \dots + P(A_n)$
15. A pair of dice is thrown. If two numbers appearing are different, find the probability that sum of the points is 8?
16. If A & B are independent s.t.  $P(A' \cap B) = 2/15$ ,  $P(A \cap B) = 1/6$ , then find  $P(B)$  ?
17. If A & B are independent events such that  $P(A \cap B) = 3/50$  &  $P(A \cup B) = 11/25$ , find  $P(A)$  &  $P(B)$  ?
18. Seeds in a certain batch have an 80% germination rate. If one plant, two seeds from this batch in the same pot, then what is the probability that :  
 (i) at least one will germinate ? (ii) exactly one will germinate?

### Determinant & Matrices

#### Group-A

( Each question carry 2 mark)

- If every element of third order determinant having value 8 is divided by 2, then what is the value of the resulting determinant ?
- if the co-factor and minor of each element of a second order determinant are same, then what is the value of the element in the second row and first column of the determinant?
- If all the elements on the two diagonals of a third order determinant are 0, then what is the value of the determinant?
- Find the value of :  $\begin{vmatrix} 03 & 3 & 101 \\ 56 & 5 & 54 \\ 21 & 7 & 23 \end{vmatrix}$  ?
- If  $\omega$  is the cube root of unity find  $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$  ?
- Can the inverse of the matrix  $\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}$  be found?



7. What is the value of  $\begin{vmatrix} \tan x & \sec x \\ \sec x & \tan x \end{vmatrix}$ ?
8. State true or false : The minor and the co-factor of the element  $a_{32}$  of a determinant of third order are equal?
9. Give an example of a unity matrix ?
10. What is  $A+B$  if  $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$  &  $B = \begin{pmatrix} 0 & -1 \\ -2 & 1 \end{pmatrix}$ ?
11. If  $\begin{vmatrix} 2 & 4 \\ k & 6 \end{vmatrix} = 0$  find the value of  $k$  ?
12. If  $\begin{vmatrix} a_1 & b_1 \\ c_1 & d_1 \end{vmatrix} = k \begin{vmatrix} a_1 & c_1 \\ b_1 & d_1 \end{vmatrix}$  ? find the value of  $k$  ?
13. Can a matrix be constructed by taking 29 elements?
14. If each element of a determinant of 3rd order is multiplied by  $k$  then how many times the value of the determinant will be multiplied?
15. Solve the equation :  $\begin{vmatrix} x+2 & x \\ 4 & 3 \end{vmatrix} = 0$  ?
16. Rectify the mistake if any. Matrix product is commutative and associative.
17. When matrix product  $AB$  and  $BA$  are defined?
18. Write true / false. "In a 3rd order determinant if there are two zeroes in a diagonal then the determinant reduced to 2nd order?"
19. Evaluate  $\begin{vmatrix} 1 & 1 \\ i & -i \end{vmatrix}$  ?
20. If  $A = \begin{bmatrix} 3 & -4 & 6 \\ 5 & 1 & 7 \end{bmatrix}$  find  $3A$  ?
21. If  $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$  find the transpose of  $A$ ?
22. Evaluate  $\begin{vmatrix} 6 & 2 & 1 \\ 9 & 3 & 2 \\ 12 & 4 & 3 \end{vmatrix}$  ?
23. Define non singular matrix ?
24. Find the value of  $\begin{vmatrix} \cos^2 \alpha & \sin^2 \alpha + 1 & 1 \\ \cos^2 \beta & \sin^2 \beta + 1 & 1 \\ \cos^2 \gamma & \sin^2 \gamma + 1 & 1 \end{vmatrix}$  ?
25. Is the multiplication of matrices always commutative?
26. What is the purpose of crammer's rule?
27. Write the number of solutions of the following system of equations  $2x+3y=7$ ,  $4x+6y=14$ ?

28. If A is a square matrix of order 3 &  $|A| = 3$ , then write the matrix represented by  $A \cdot \text{adj } A$ ?

29. If  $\omega$  is the cube root of unity find  $\lambda$  for which  $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \lambda & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0$ ?

30. If  $[1 \ 2 \ 3]A = [0]$ , then what is the order of the matrix A?

31. Write the value of k if:  $\begin{vmatrix} aa_1 & aa_2 & aa_3 \\ ab_1 & ab_2 & ab_3 \\ ac_1 & ac_2 & ac_3 \end{vmatrix} = k \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ ?

### Group-B

(Each question carry 5 mark)

1. Without expanding find the value of  $\begin{vmatrix} 3 & 6 & 9 \\ -2 & 4 & -6 \\ 8 & 16 & 24 \end{vmatrix}$ ?

2. Matrices X & Y are s.t.  $3X + 4Y = I$ ,  $X - 2Y = 2I$ , I is the unity matrix  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Determine the matrix X.

3. Find the inverse of  $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ ?

4. Evaluate:  $\begin{vmatrix} \sin^2 \theta & \cos^2 \theta & 1 \\ \cos^2 \theta & \sin^2 \theta & 1 \\ -10 & 12 & 2 \end{vmatrix}$ ?

5. Find x & y if  $\begin{pmatrix} 3 & 2 \\ 7 & x \end{pmatrix} \begin{pmatrix} 5 & -2 \\ -7 & y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ?

6. Prove that  $\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a & 1 \end{vmatrix}$  is a perfect square?

7. If  $\begin{pmatrix} 4x & ax - 2u \\ 2u + v & 3v - 2w \end{pmatrix} = \begin{pmatrix} 8 & 6 \\ 3 & 5 \end{pmatrix}$ , find x, u, v, w?

8. Solve by crammer's rule:  $2x - 3y = -1$ ,  $3x + 2y = 5$ ?

9. If  $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 5 & 1 \end{bmatrix}$ , find  $A^2 B$ ?

10. Evaluate using properties of determinants:  $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 4 & 4 & 6 \end{vmatrix}$

11. Find x, y when  $\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ ?



12. Solve :  $\begin{vmatrix} p & q & r \\ p & q & x \\ x & q & r \end{vmatrix} = 0$ , given  $p, q, r \neq 0$  ?

13. Construct square matrices A & B of order 2 s.t.  $AB=BA$  but each is not equal to unit matrix ?

14. If A is  $2 \times 2$  non singular matrix and  $|A| = 1/2$  then which matrix are represented by  $A \cdot \text{Adj}A$ ?

15. Find the value of  $\begin{vmatrix} -\csc^2 \theta & \sec^2 \theta & -0.2 \\ \cot^2 \theta & -\tan^2 \theta & 1.2 \\ -1 & 1 & 1 \end{vmatrix}$  ?

16. Write the following in matrix form :  $2x-3y+2=0$ ,  $x-4=0$  ?

17. If  $a_{ij}=i-j$  then form a matrix of  $3 \times 3$  whose elements are  $a_{ij}$  ?

18. Find x so that  $(1 \times 1) \begin{pmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ x \end{pmatrix} = 0$ ?

19. If the system of equations  $\lambda x + 3y = 0$ ,  
 $x + (\lambda - 2)y = 0$  has infinitely many solutions, then find the value of  $\lambda$  ?

20. Prove that :  $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \Rightarrow A^2 - 5A + 7I = 0$  ?

21. Test whether the following system of equations have non zero solution. Write the solution set:  $2x+3y+4z=0$ ,  $x-2y-3z=0$ ,  $3x+y-8z=0$  ?

**Group-C** (Each question carry 10 mark)

22. Without expanding prove  $\begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix} = \begin{vmatrix} y & b & a \\ x & a & p \\ z & c & r \end{vmatrix}$

23. Prove that  $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

24. Find the inverse of  $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & 0 & 2 \end{pmatrix}$  ?

25. Solve  $3x+2y=12$ ,  $5x-3y=1$  by matrix method?

26. Prove that  $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$  ?

27. Prove  $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$  ?

28. Solve by matrix inversion method :  $3x-2y+z=1$ ,  $2x+y-5z=2$ ,  $x+y-2z=3$  ?

29. Find the inverse of  $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$ ?

30. Solve by crammer's rule :  $x+y+2z=4$ ,  $2x-y-2z=-1$ ,  $3x-2y-2z=1$  ?

31. Prove :  $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left( 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$ ?

32. If  $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ , find  $A^{-1}$ ?

33. Solve by crammer's rule :  $5x+2y=4$ ,  $7x+3y=5$ ?

34. Solve by crammer's rule :  $x+2y+z=2$ ,  $2x-y-z=2$ ,  $2x-3y-2z=1$  ?

35. Solve by matrix method :  $2x+y-3z=0$ ,  $x-y+z=4$ ,  $x+y+z=2$  ?

36. Prove  $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$ ?

37. Prove  $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$ ?

38. If  $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$  find  $(AB)^{-1}$  ?

39. Solve by matrix inversion method :  $x-2y=3$ ,  $3x+4y-z=-2$ ,  $5x-3z=-1$  ?

40. If  $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  find  $k$  so that  $A^2=kA-2I$  ?

41. Evaluate  $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ ?

42. For two matrices  $A = \begin{bmatrix} 1 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$ . Verify that  $(AB)^T = B^T A^T$  ?

43. If  $a, b, c$  are real numbers then prove that the minimum value of the determinant

$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix}$  is 1?

44. Prove that  $\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} = 4xyz$  ?

45. Find the adjoint of the matrix :  $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ ?



46. Prove :  $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a^2+b^2+c^2)(a+b+c)(a-b)(b-c)(c-a) ?$

47. Solve the following by matrix inversion method:  $x-y+z=4$ ,  $2x+y-3z=0$ ,  $x+z+z=2$  ?

48. Prove  $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^2 ?$

### Linear programming problem

#### Group-A

( Each question carry 1 mark)

1. Mention the quadrant in which the solution of an LPP with two decision variables lies when the graphical method is adopted?
2. Define feasible solution of LPP.
3. Write the solution of the following LPP.

$$\text{Max} Z = 2x + 3y$$

$$\text{Subject to } x + y \leq 1, x \geq 0, y \geq 0$$

4. Write the maximum value of  $x+y$  subject to  $2x+3y \leq 6$ ,  $x \geq 0, y \geq 0$ ?

#### Group-B

( Each question carry 2 mark)

1. Let an LPP as follows :

$$\text{Maximize : } z = 3x_1 + 5x_2$$

$$\text{Subject to } 5x_1 + 3x_2 \leq 30, x_1 + 2x_2 \leq 12, 2x_1 + 5x_2 \leq 20, x_1, x_2 \geq 0$$

Test whether the points (2,3) & (-3, 4) are feasible solutions or not ?

2. Show the feasible region of the following constraints in graph:  $2x + y \leq 4, x \geq 0, y \geq 0$ ?
3. Shade the feasible region for the inequation:  $3x + 2y \leq 6, x \geq 0, y \geq 0$ ?
4. Shade the feasible region for the inequation:  $2x + 3y \leq 6, x \geq 0, y \geq 0$  in a rough sketch?

( Each question carry 6 mark)

#### Group-C

7. Solve graphically the following LPP :

$$\text{Maximize : } z = 3x_1 + 2x_2$$

$$\text{Subject to : } -2x_1 + x_2 \leq 1, x_1 \leq 2, x_1 + x_2 \leq 3, x_1, x_2 \geq 0 ?$$

8. Solve graphically the following LPP :

$$\text{Max} Z = 2x + 3y$$

$$\text{Subject to } x + y \leq 400, 2x + y \leq 600 \text{ and } x, y \geq 0$$

9. Solve graphically the following LPP :

$$\text{Max} Z = 5x + 3y$$

$$\text{Subject to } 3x + 5y \leq 15, 5x + 2y \leq 10 \text{ and } x, y \geq 0$$

### 3D & Vectors

#### Group - A

( Each question carry 2 mark)

1. What is the image of a point (6,3,-4) w.r.t. yz-plane?

2. Find  $k$  so that the line  $\frac{x-2}{3} = \frac{1-y}{k} = \frac{z-1}{4}$  is parallel to the plane  $2x+6y+3z-4=0$ ?
  3. To which co-ordinate axes is the plane  $2x+3z=0$  parallel and why?
  4. Write the values of  $a, b, h$  for which the equation  $ax^2 + 2hxy + by^2 + 3z^2 - 4x - 6y - 4 = 0$  represents a sphere?
  5. Write the equation of the plane passing through the point  $(1, -2, 3)$  and perpendicular to  $y$ -axis?
  6. How many independent constant are there in the general equation of a plane  $ax+by+cz+d=0$ ?
  7. If  $\vec{a} \cdot \vec{b} = 0$  &  $\vec{a} \times \vec{b} = \vec{0}$  then draw the conclusion?
  8. If  $A, B, C, D$  are the vertices of a regular pentagon, find  $\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} + \vec{EA}$ ?
  9. How many directions of a null vector has?
  10. For what value of  $\lambda$  the vectors  $\lambda\hat{i} + 3\hat{j} + \lambda\hat{k}$  and  $\lambda\hat{i} - 2\hat{j} + \hat{k}$  are perpendicular to each other?
  11. Write the values of  $a$  and  $b$  for which the vectors  $(a-1)\hat{i} + (b+2)\hat{j} + 4\hat{k}$  and  $(a+1)\hat{i} + (b-2)\hat{j} + 8\hat{k}$  will be parallel?
  12. Determine the values of  $\mu$  for which the vector  $\vec{\alpha} = \mu(6\hat{i} + 2\hat{j} - 3\hat{k})$  will be of unity length?
  13. Write a vector normal to  $\hat{i} + \hat{k}$  and  $\hat{i} + \hat{j}$ ?
  14. Write the values of  $m$  and  $n$  for which the vectors  $(m-1)\hat{i} + (n+2)\hat{j} + 4\hat{k}$  and  $(m+1)\hat{i} + (n-2)\hat{j} + 8\hat{k}$  will be parallel?
- Group - B** (Each question carry 5 mark)
15. Find the equation of the plane bisecting the line segment joining  $(-1, 4, 3)$  and  $(5, -2, -)$  at right angles?
  16. If one end of the diameter of the sphere  $x^2 + y^2 + z^2 - 2x + 4y - 6z - 11 = 0$  is  $(-1, 2, 4)$  find the co-ordinate of the other end?
  17. Determine the point on the sphere  $x^2 + y^2 + z^2 - 6x - 8y + 4z = 0$  diametrically opposite to the origin?
  18. Find the image of the point  $(-2, 0, 3)$  w.r.t. the plane  $y = 3$ ?
  19. Find the equation of the plane passing through the line  $x = y = z$  and the point  $(3, 2, 1)$ ?
  20. Prove that the straight line  $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-3}{1}$  lies on the plane  $7x+5y+z=0$ ?
  21. If the position vectors of the points  $A, B, C$  are  $2\hat{i} + \hat{j} - \hat{k}, 3\hat{i} - 2\hat{j} + \hat{k}, \hat{i} + 4\hat{j} - 3\hat{k}$  respectively, then prove that  $A, B, C, D$  are collinear?
  22. Find the equation of the plane that passes through  $(-1, 3, 0)$  and its perpendicular to the line through  $(1, 1, 1)$  and  $(2, -1, -2)$ ?



23. The line segment joining the points (4,5,-6) and (2,3,4) is a diameter of the sphere, find equation of the sphere?
24. Find the scalar projection of the vector  $\vec{a} = 3\hat{i} + 6\hat{j} + 9\hat{k}$  on  $\vec{b} = 2\hat{i} + 2\hat{j} - \hat{k}$ ?
25. Find the value of  $\lambda$  s.t. the following vectors are co-planar:  
 $-\hat{i} + \lambda\hat{j} - \lambda\hat{k}, 2\hat{i} + 4\hat{j} + 5\hat{k}, -2\hat{i} + 4\hat{j} - 4\hat{k}$ ?
26. Find the equation of the sphere with its centre at (1, -2, 3) and touching the plane  $2x - 3y + z + 6 = 0$ ?
27. If A (1, 0, -2), B (-2, 4, -2) & C (1, 5, 10) be the vertices of a triangle and the bisector of the angle BAC meets BC at D then find the co-ordinates of the point D?
28. If  $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} \neq 0$  prove  $\vec{a} + \vec{c} = m\vec{b}$ ; m is a scalar?
29. Find the component of the vector  $\vec{b} = 8\hat{i} + \hat{j}$  in the direction of  $\vec{a} = \hat{i} + 2\hat{j} - 2\hat{k}$ ?
30. Show that the vectors  $2\hat{i} + 3\hat{j}, 5\hat{i} - 5\hat{k}, 6\hat{j} + 4\hat{k}$  are co-planar?
31. Find the volume of parallelopiped whose coterminous edges are  $2\hat{i} - \hat{j} + \hat{k}, \hat{i} + 2\hat{j} - \hat{k}, \hat{j} + \hat{k}$ ?
32. Resolve the vector  $\vec{b} = \hat{i} + \hat{j} + \hat{k}$  into vectors parallel and perpendicular to the vector  $\vec{a} = \hat{i} + \hat{j}$ ?
33. Find the area of the parallelogram whose diagonals are the vectors  $3\hat{i} + \hat{j} - 2\hat{k}$  and  $\hat{i} - 3\hat{j} + 4\hat{k}$ ?
34. Determine the value of m for which the following vectors are orthogonal:  
 $(m+1)\hat{i} + m^2\hat{j} - m\hat{k}$  and  $\vec{a} = (m^2 - m + 1)\hat{i} - m\hat{j} + \hat{k}$ ?

#### Group-C

( Each question carry/10 mark)

1. Show that the lines  $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$  &  $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$  intersect. Find the co-ordinates of the point of intersection of the lines and the plane in which the lines lie?
2. A variable plane is at a constant distance 3r from the origin and meets the axes in A, B, C. Show that the locus of the centroid of the triangle ABC is  $x^2 + y^2 + z^2 = r^2$ ?
3. Find the equation of the sphere inscribed in the tetrahedron whose faces are  $x=0, y=0, z=0, 2x+2y+z=0$ ?
4. Find the distance of the point (1, -2, 3) from the plane  $x-y+z=5$  measured parallel to the line  $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ ?
5. If the vertices A, B, C of a triangle ABC are at (1, 1, 2), (2, 2, 3), (3, -1, -1) respectively, then using vector method find the area of the triangle?
6. A sphere passes through the point (5, 1, -4) and (1, -2, 5). If the centre lies on y-axis find its equation and determine whether the origin lies inside or outside the sphere?

7. Prove that four points  $(0,4,3), (-1,-5,-3), (-2,-2,1)$  and  $(1,1,-1)$  lie in one plane. Find equation of the plane?
8. Prove that the lines  $x+5y+2z+3=0=3x+2y+z-2$  and  $\frac{x+5}{3}=\frac{y+4}{1}=\frac{z-7}{-2}$  are coplanar. Find the point of intersection and the equation of the plane in which they lie?
9. A sphere touches the plane  $x+2y+2z=5$  at  $(1,1,1)$  and the plane  $2x+2y+z=39$  at  $(7,9,7)$ . Find the equation of the sphere. Decide whether the origin is inside this sphere?
10. For  $\vec{a}=\vec{i}-\vec{j}, \vec{b}=\vec{i}-2\vec{k}, \vec{c}=\vec{j}+\vec{k}$ , obtain  $\vec{a} \times (\vec{b} \times \vec{c})$  and verify the formula  $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$ ?
11. If  $\vec{a}=\vec{i}+\vec{j}, \vec{b}=-\vec{i}+2\vec{k}, \vec{c}=\vec{j}+\vec{k}$ , then verify the formula  $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$ ?
12. Prove that:
- $[\vec{p}-\vec{q}, \vec{q}-\vec{r}, \vec{r}-\vec{p}]=0$
  - $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$

.....End.....

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